Design Goals and Programming Paradigms

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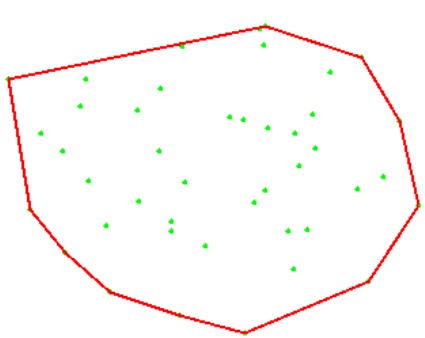
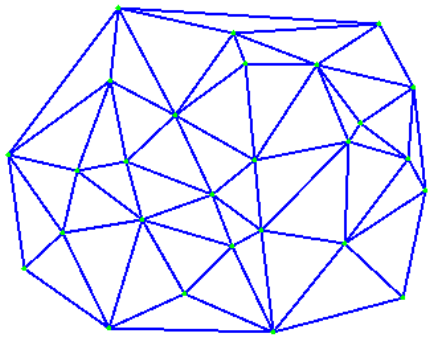
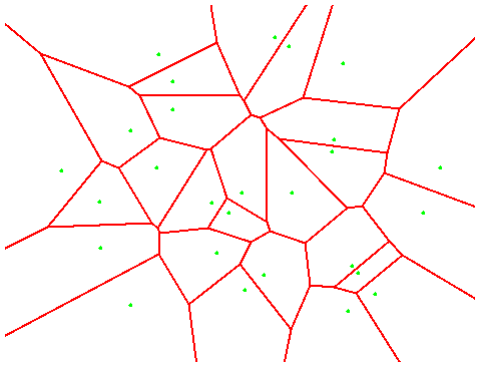
Outline

Examples of Computational Geometry Algorithms

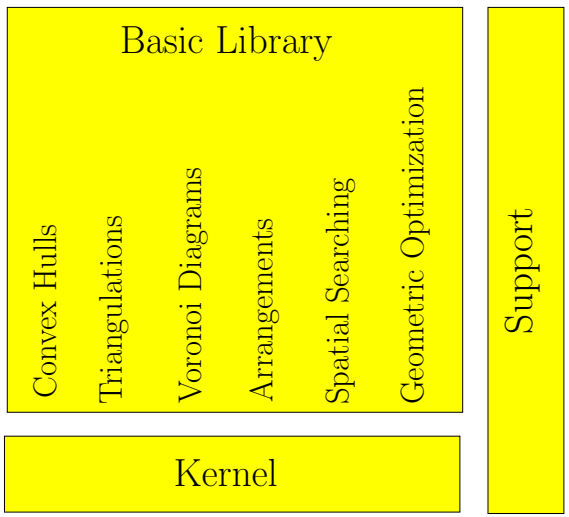
- Convex hulls, triangulations, Voronoi diagrams

- Emphasis on asymptotic complexity (Real-RAM model)
- Algorithms manipulating large numbers of geometric objects

CGAL: Software Architecture

General architecture: kernel, basic library, support library



Kernel of geometric primitives

Algorithms are logically split into:

- a combinatorial part (constructs a graph)
- a numerical part (makes use of coordinates)

The latter calls primitives gathered in the kernel:

- Basic objects: points, segments, lines, circles...
- Predicates: orientations, abscissa comparisons...
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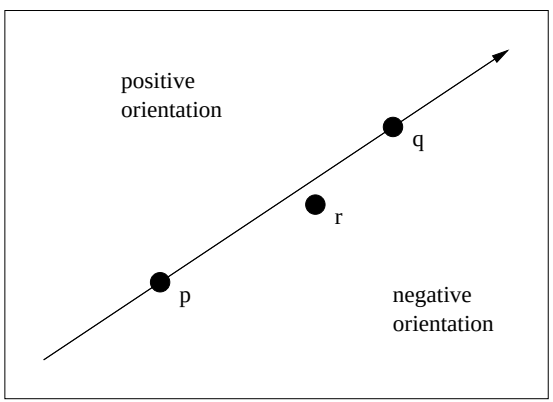
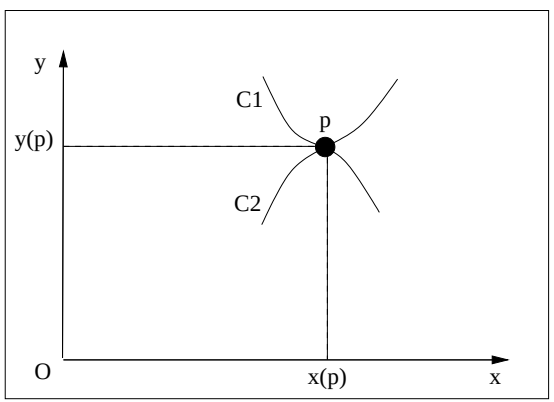
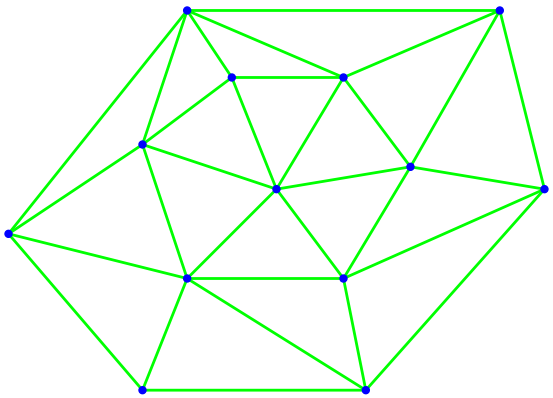




Example: Delaunay triangulation

Incremental algorithm in 2 steps: point location and update.



Point location: orientation(p, q, r) predicate, sign of:

$$\begin{vmatrix} 1 & px & py \\ 1 & qx & qy \\ 1 & rx & ry \end{vmatrix} = \begin{vmatrix} qx - px & qy - py \\ rx - px & ry - py \end{vmatrix}$$

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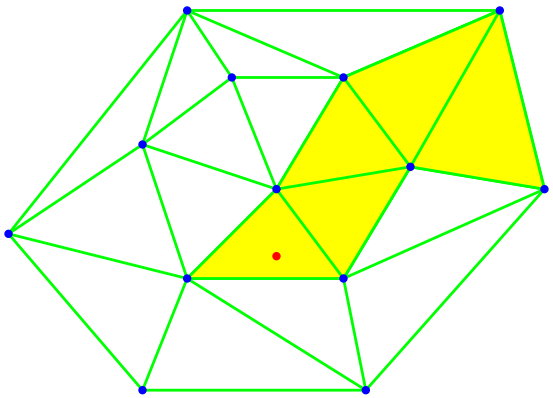

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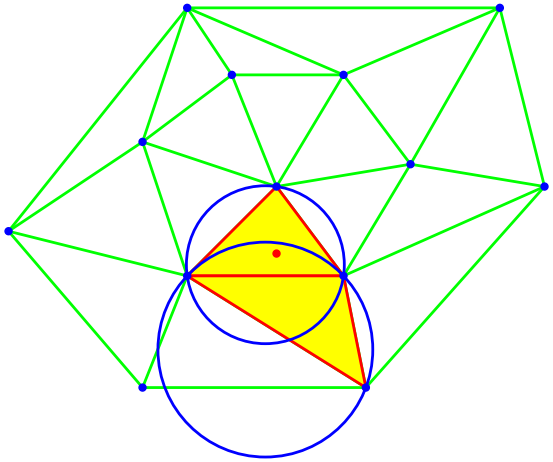

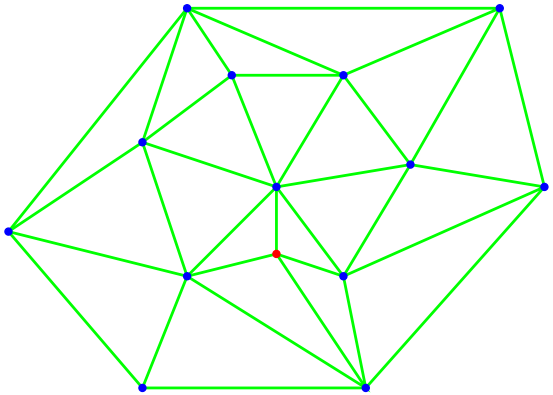

Update: in_circle(p, q, r, s) predicate, sign of:

$$\begin{vmatrix} 1 & px & py & px^2+py^2 \\ 1 & qx & qy & qx^2+qy^2 \\ 1 & rx & ry & rx^2+ry^2 \\ 1 & sx & sy & sx^2+sy^2 \end{vmatrix}$$

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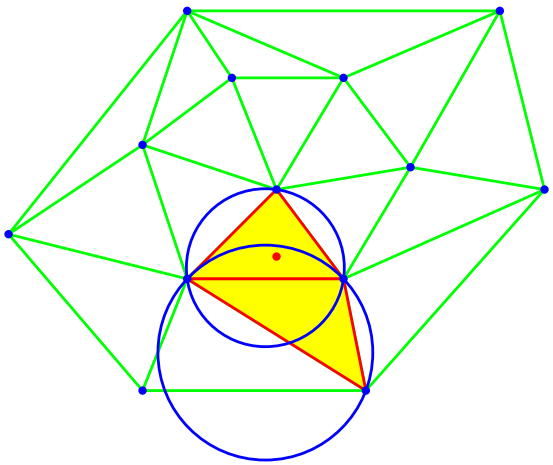

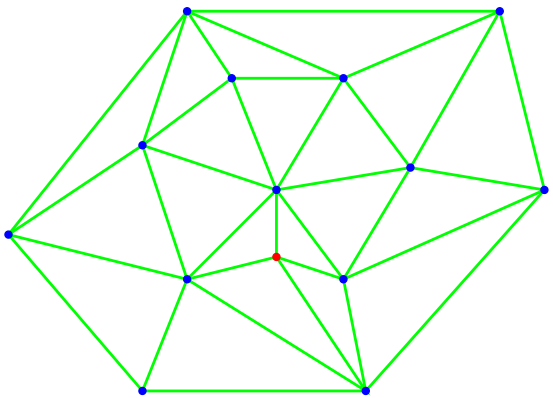

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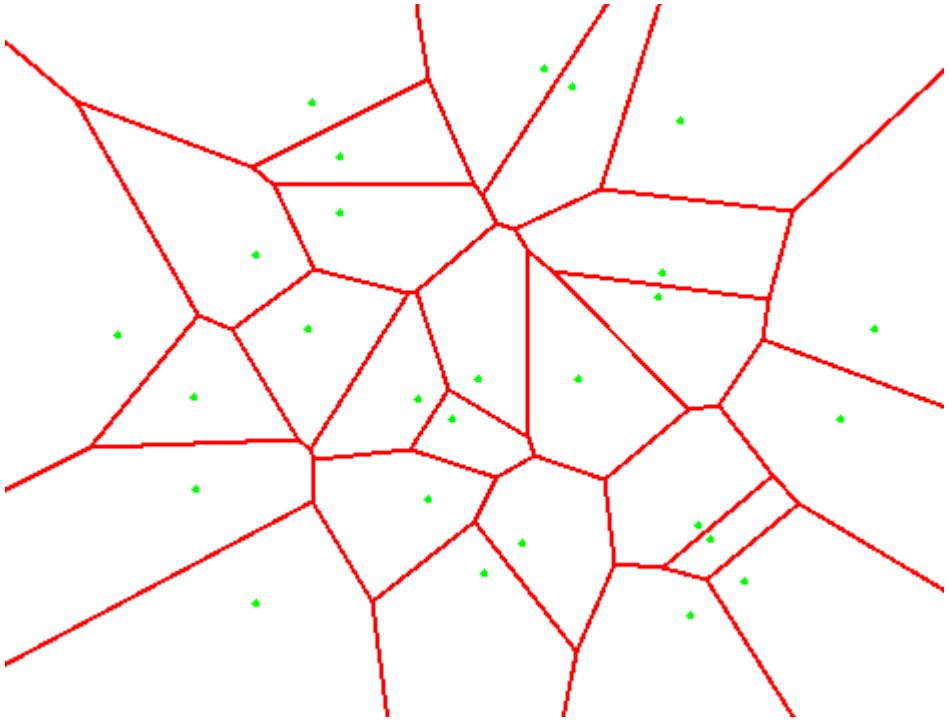
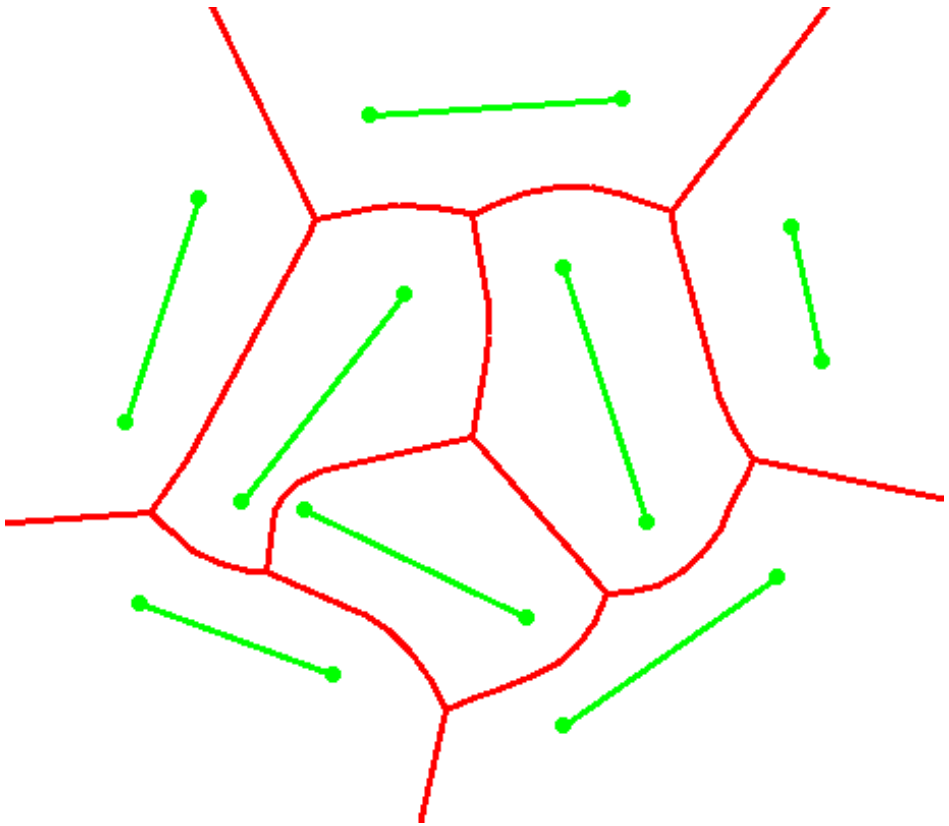
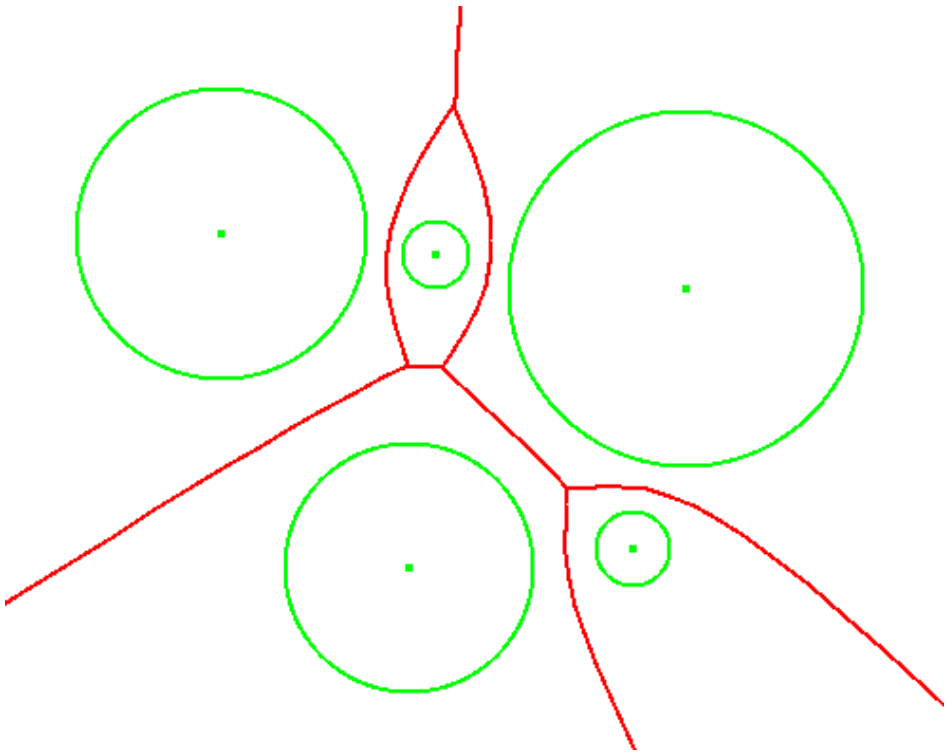
Voronoi diagram of points



Voronoi diagram of line segments

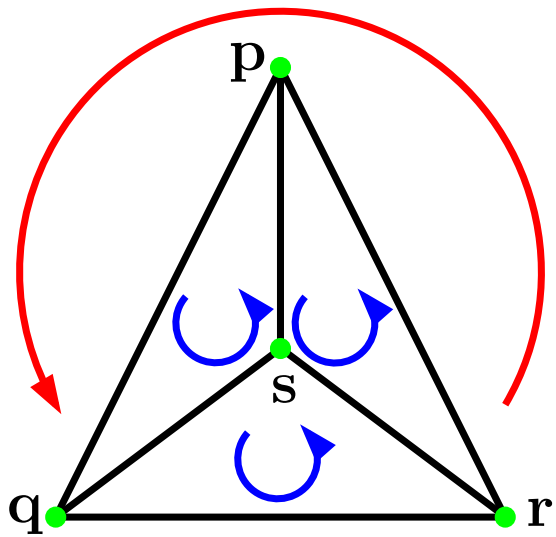


Voronoi diagram of circles



Robustness problems

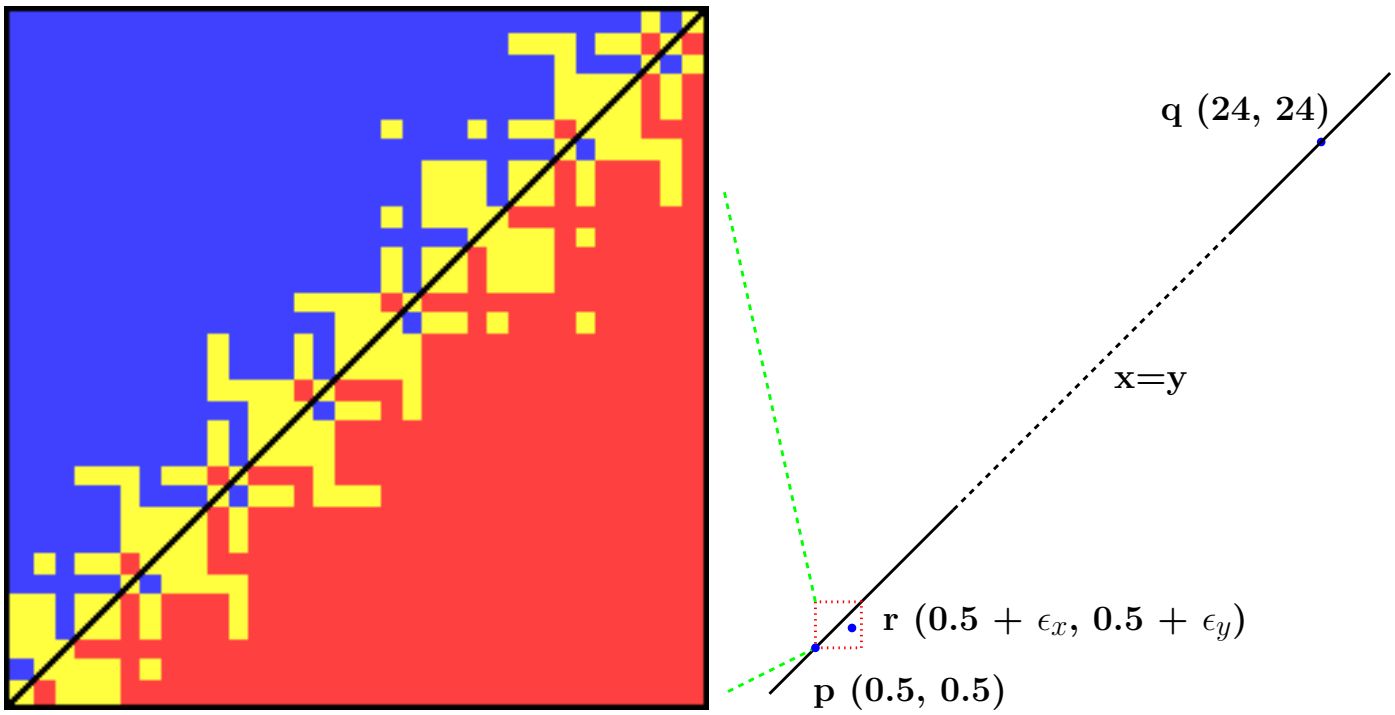
Algorithms rely on mathematical theorems, like:



ccw(s, q, r)
ccw(p, s, r) => ccw(p, q, r)
ccw(p, q, s)

Robustness

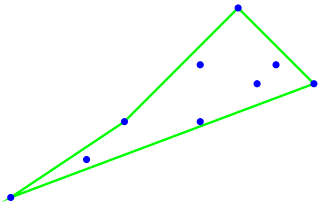
Example where floating-point geometry differs from real geometry:
orientation of almost collinear points.

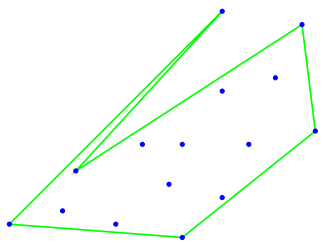


[Kettner, Mehlhorn, Schirra, P., Yap, ESA'04]

Potential consequences on algorithms

- Slightly wrong result





- Completely wrong result
- The algorithm stops on a invalid assertion
- The algorithm loops forever

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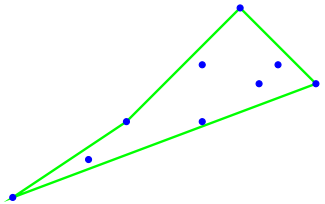

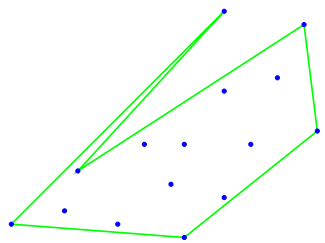

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Robustness: solutions

- Case-by-case treatment: painful, error prone, not mathematically nice
- Use exact predicates (*Exact Geometric Computing*)

Remarks

- Floating-point fails on [almost] degenerate cases.
- These cases happen more of less often in practice.

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Arithmetic: Number types

Geometric primitives are parameterized by the arithmetic.

- Multiple precision integers [GMP, MPFR, LEDA...]
- Multiple precision rationals
- Multiple precision floating-point
- Interval arithmetic (with `double` or MP bounds)

Algebraic numbers:

- Numerical evaluation with separation bounds [CORE, LEDA]
- Polynomials, Sturm, resultants... [CGAL, SYNAPS]

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Filtered predicates

Speed up exact predicates using a filter:

- floating-point evaluation with a certificate
- multiple precision only when needed

Examples

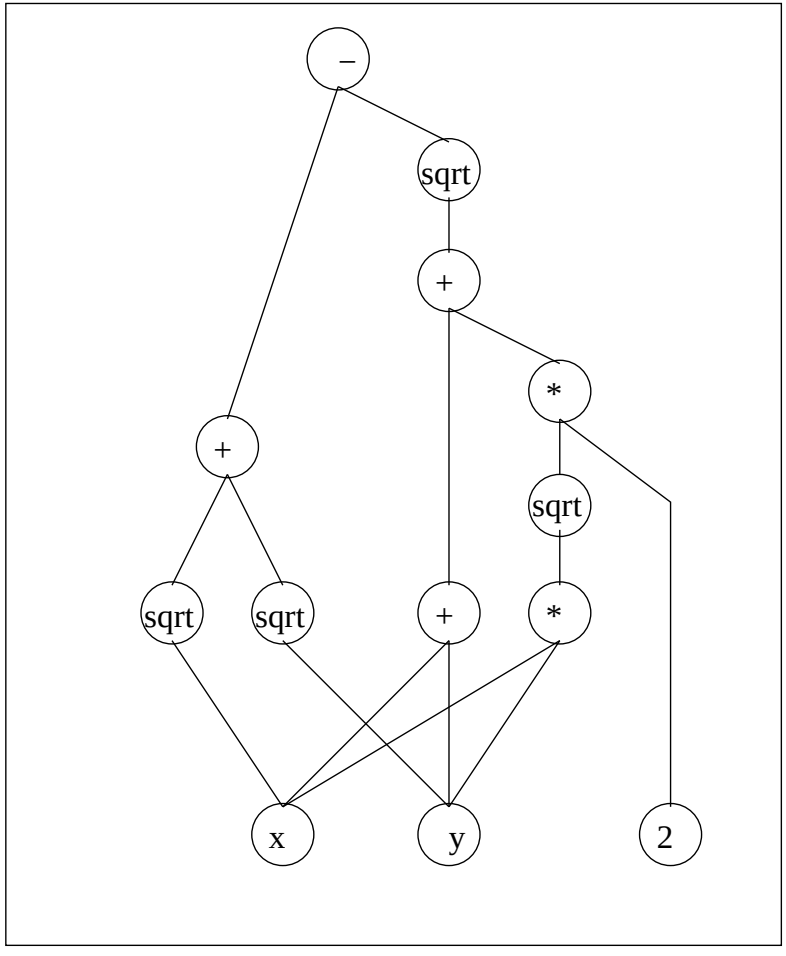
- interval arithmetic (dynamic filters), [Burnikel, Funke, Seel – Brönnimann, Burnikel, P.'98]
- or static code analysis (static filters) [Fortune'93... Melquiond, P.'05]

Programming aspects:

- automatic generation of filtered predicates
- cascading/pipelining various methods

Filtered number types

DAG of the operations in memory, e.g.: $\sqrt{x} + \sqrt{y} - \sqrt{x + y + 2\sqrt{xy}}$



Approximation and iterative precision refinement, on demand.

Filtered predicates: benchmarks

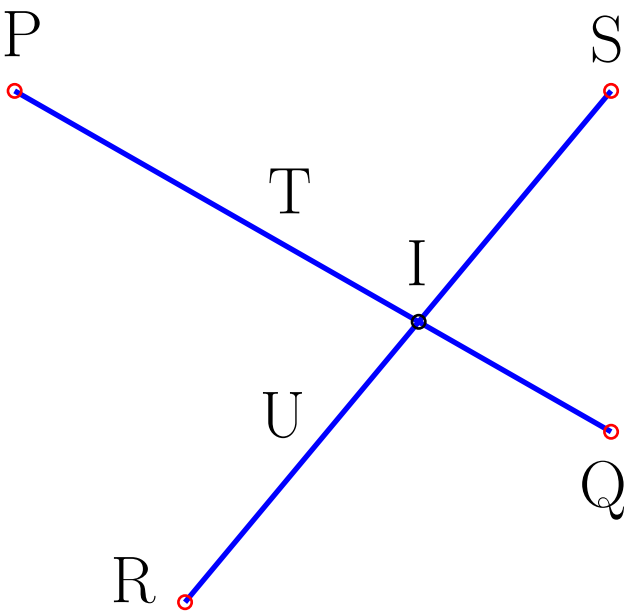
Running time of a 3D Delaunay triangulation.

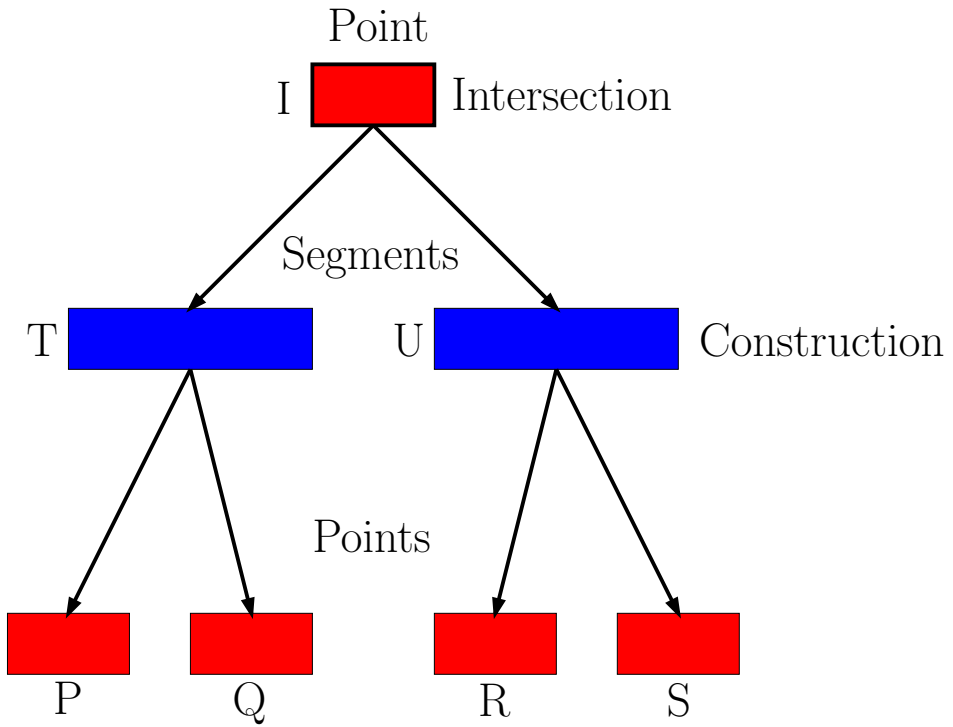
	R5	E	M	B	D
double	40.6	41.0	43.7	50.3	`loops`
MPF	3,063	2,777	3,195	3,472	214
Interval + MPF	137.2	133.6	144.6	165.1	15.8
semi static + Interval + MPF	51.8	61.0	59.1	93.1	8.9
almost static + semi static + Interval + MPF	44.4	55.0	52.0	87.2	8.0
Shewchuk's predicates	57.9	57.5	62.8	71.7	7.2
CORE Expr	570	3520	1355	9600	173
LEDA real	682	640	742	850	125
`Lazy_exact_nt<MPF>`	705	631	726	820	67

An important criteria: failure rate of filters.
User interface in CGAL: choice of different kernels.

Filtered geometric constructions

Additional difficulty: storage of geometric objects in memory
Bonus: regrouping computations, and less memory





Algorithms and traits classes

Generic programming: algorithms are decoupled from the data structures and the numerics. Similarly to the C++ STL.

Algorithms are parameterized (templates) by geometric traits classes which provide:

- types of the objets manipulated by the algorithm: Point_2, Tetrahedron_3...
- predicates that the algorithm applies to objets: Orientation_2, Side_of_oriented_sphere_3...
- constructions : Construct_mid_point_2, Construct_circumcenter_3, Compute_squared_length_2...

These last two are provided under the form of function objects.
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Kernel interface

CGAL geometric algorithms are parameterized by a geometric traits.

```
template < class TriangulationTraits_3 ,
           class TriangulationDataStructure_3 = ... >
class Triangulation_3 ;
```

It provides a (attempted minimal) set of types and functions for the geometry :

Nested Type	Requirements
Point_3	Assignable, DefaultConstructible
Segment_3	Assignable, DefaultConstructible
Orientation_3	Function object taking 4 Point_3, returning enum...
Circumcenter_3	...

The Kernel concept is a superset for many different geometric traits concepts. There are many different possible (and useful) models of Kernel.

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Kernels

The kernel can be used as a model for the geometric traits class parameter in numerous algorithms.

Basic kernels, parameterized by number types:
Cartesian<FT>
Homogeneous<RT>

Ex : Triangulation_3<Cartesian<double> >

Cartesian<double> is a model of the TriangulationTraits_3 concept.

Kernel functionality is also available as free functions:

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Valid parameters for the Cartesian kernel...

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Multiple precision :

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Number types that include filtering:

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Filtered kernels

Internal tools:
Interval arithmetic: CGAL::Interval_nt, boost::interval

Filtered predicates generator using C++ exceptions: CGAL::Filtered_predicate<>

CGAL::Filtered_kernel< *K* > provides some predicates based on static filtering, and all others dynamic (IA).

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Recommended kernels:
CGAL::Exact_predicates_exact_constructions_kernel
CGAL::Exact_predicates_inexact_constructions_kernel

Example 1

```
template < typename K >
struct My_orientation_2
{
  typedef typename K::RT       RT;
  typedef typename K::Point_2  Point_2;

  CGAL::Orientation
  operator()(const Point_2 &p, const Point_2 &q,
             const Point_2 &r) const
  {
    RT prx = p.x() - r.x();   RT pry = p.y() - r.y();
    RT qrx = q.x() - r.x();   RT qry = q.y() - r.y();
    return static_cast<CGAL::Orientation> (
             CGAL::sign( prx*qry - qrx*pqy ) );
  }
};
```

Example 2

```
// We use it:
typedef CGAL::Cartesian<double>  Kernel;
Kernel::Point_2 p(1, 2), q(2, 3), r(4, 5);
My_orientation_2<Kernel> orientation;
CGAL::Orientation ori = orientation(p, q, r);
```

Using `Filtered_predicate`

```
typedef CGAL::Simple_cartesian<double> K;
typedef CGAL::Simple_cartesian<CGAL::Interval_nt_advanced> FK;
typedef CGAL::Simple_cartesian<CGAL::MP_Float> EK;
typedef CGAL::Cartesian_converter<K, EK> C2E;
typedef CGAL::Cartesian_converter<K, FK> C2F;

typedef CGAL::Filtered_predicate<My_orientation_2<EK>,
                                 My_orientation_2<FK>,
                                 C2E, C2F>  Orientation_2;

...
  K::Point_2 p(1,2), q(2,3), r(3,4);
  Orientation_2 orientation;
  orientation(p, q, r);
  return 0;
```

C++ is not everything: what about other languages?

We have started to write CGAL wrappers in other languages:

- Scilab, Matlab: not object-oriented, complete rethinking needed
- Python, Java: object-oriented, similar interface and paradigms

Python interface

Python is:

- object-oriented (syntax similar to C++)
- interpreted (precompiled, no templates)
- general purpose, used more and more for scientific tasks
- C++ wrappers generators exist
- cgal-python has already wrapped several CGAL packages
  `http://cgal-python.gforge.inria.fr/`

Java interface

Java is:

- as important as C++
- simpler (garbage collection...)
- syntax is in the same family as C++
- status: the 2D triangulation class has been wrapped.

Conclusion

Thank you for your attention.